

Using the limit definition of the definite integral, and right endpoints, evaluate $\int_{-1}^3 (3x+1) dx$.

SCORE: ____ / 15 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\begin{aligned} & \textcircled{1} \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 \left(-1 + \frac{4i}{n} \right) + 1 \right) \frac{4}{n} \textcircled{2} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left(-2 + \frac{12i}{n} \right) \textcircled{2} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left(\sum_{i=1}^n -2 + \frac{12}{n} \sum_{i=1}^n i \right) \textcircled{1} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n} \left(\underbrace{-2n}_{\textcircled{1}} + \frac{12}{n} \cdot \frac{n(n+1)}{2} \right) \textcircled{1} \\ &= \lim_{n \rightarrow \infty} \left(-8 + \frac{24(n+1)}{n} \right) \textcircled{1} \\ &= -8 + \frac{24}{1} \textcircled{1} \\ &= 16 \textcircled{1} \end{aligned}$$

① FOR WRITING
 $\lim_{n \rightarrow \infty}$
ON EVERY LINE

A person's velocity as a function of time is shown in the graph on the right.

NOTE: The graph is a piecewise defined function whose "pieces" are all linear functions.

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- [a] Estimate the distance the person travelled from time $t = 2$ second to $t = 10$ seconds using four subintervals and left endpoints.

$$\Delta t = \frac{10-2}{4} = 2$$

$$v(2)\Delta t + v(4)\Delta t + v(6)\Delta t + v(8)\Delta t$$

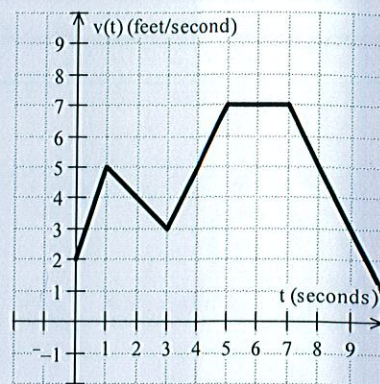
$$= 4 \cdot 2 + 5 \cdot 2 + 7 \cdot 2 + 5 \cdot 2 = 42 \text{ ft}$$

- [b] Find the exact distance the person travelled from time $t = 2$ second to $t = 10$ seconds.

$$\frac{1}{2}(4+3)(1) + \frac{1}{2}(3+7)(2) + 7(2) + \frac{1}{2}(7+1)(3)$$

$$= \frac{7}{2} + 10 + 14 + 12 \quad \textcircled{1} \text{ POINT EACH}$$

$$= 39\frac{1}{2} \text{ ft}$$



The graph of function f is shown on the right.

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The graph consists of a horizontal line, a semi-circle with center $(-2, 1)$, a diagonal line and a horizontal line.

- [a] Evaluate $\int_{-5}^1 f(x) dx$.

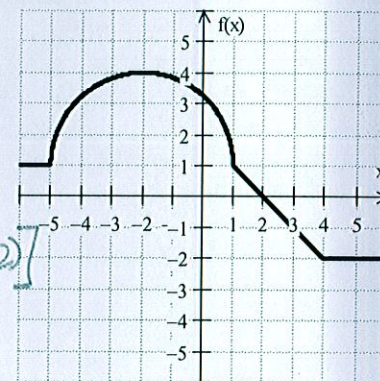
$$= -\int_1^5 f(x) dx$$

① POINT EACH

$$= -\left[\int_1^2 f(x) dx + \int_2^5 f(x) dx\right] = -\left[\frac{1}{2}(1)(1) + \frac{1}{2}(3+1)(2)\right]$$

- [b] Evaluate $\int_{-4}^5 f(x) dx$.

$$= -\left[\frac{1}{2} - 4\right] = \frac{7}{2}$$



Evaluate $\int_0^4 (|x-3| + \sqrt{16-x^2}) dx$ using the properties of definite integrals

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and interpreting in terms of area.

$$= \int_0^4 |x-3| dx + \int_0^4 \sqrt{16-x^2} dx = \frac{1}{2}(3)(3) + \frac{1}{2}(1)(1) + \frac{1}{4}\pi(4)^2$$

$$= \frac{9}{2} + \frac{1}{2} + 4\pi$$

$$= 5 + 4\pi$$

① POINT EACH

